**Seat No. _________________** **Enrollment No._________________**

# C. U. SHAH UNIVERSITY

## M.Sc. (Mathematics) Semester-II Summer – 2015

**Subject Name: Algebra-I Subject Code: 5SC02MTC5**

**Time: 03 hours** **Maximum Marks: 70**

Instructions:
1. Attempt all questions.
2. Make suitable assumption whenever necessary.
3. Figures to the right indicate full marks.

**Section– I** Marks

Q-1 a) Define : Principal Ideal , Principal Ideal Ring (02)

b) Define : Euclidean ring (02)

c) Let R be a ring with identity , I be an ideal in R. Prove if $1 \in R$ then I = R. (02)

d) Define : Associates. (01)

Q-2 a) Let R be a Euclidean ring. $a_0 \in R.$ Prove that the ideal $A = \langle a_0 \rangle$ is maximal if and only if $a_0$ is prime element of R. (07)

b) Let R be a Euclidean ring, then any two elements a and b in R have a greatest common divisor d = (a , b) in R. Moreover d = λa + μb for some λ , μ in R. (04)

c) Let $I = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} / a, b \in Z \right\}$. Is it an ideal of $M_2(Z)$? Justify your answer. (03)

OR

Q-2 a) Prove that the Gaussian ring of integers J[i] is a Euclidean ring. (07)

b) Let R be a Euclidean ring, then any element in R is either a unit in R or can be written as a product of a finite number of prime elements in R. (04)

c) Prove that every Euclidean ring has identity. (03)

Q-3 a) Let $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ be a polynomial with integer coefficients. Suppose that for some prime number p $,p/a_0, p/a_1 , \dots , p/a_{n-1}$but p does not divide $a_n$ and $p^2$ does not divide $a_o$. Then prove $f(x)$ is irreducible over Q. (05)

b) State and prove Unique Factorization Theorem. (05)

c) Let R be a Euclidean ring. Then prove for any a,b,c in R if (a , b) = 1 and $a/bc$ then $a/c$ . (04)

OR

Q-3 a) Prove that if $f(x)$ and $g(x)$ in $Z[x]$ are primitive then $f(x). g(x)$ is also primitive polynomial. (05)

b) Let $f(x)$ and $g(x)$ are two nonzero elements of F[x], F is a field then (05) prove that $\deg(f(x).g(x)) = \deg(f(x)) + \deg(g(x))$.

c) Let R be a Euclidean ring. Prove if π is a prime element in R and $\pi/ab$ then $\pi/a$ or $\pi/b$ . (04)

## Section – II

Q-4 a) Define : Irreducible polynomial , Primitive polynomial. (02)

b) Define : Galois group. (02)

c) Define: Radical extension. (02)

d) Define: Commutator of a and b in a group $G$. (01)

Q-5 a) Prove that following polynomials are irreducible over Q (07)
(1) $f(x) = x^2 - 12$ (2) $f(x) = 3x^4 - 4x^2 + 8x + 10$
(3) $f(x) = 3x^4 - 7x^3 + 14$

b) $\sqrt{2} + \sqrt{3}$ is algebraic over Q ? If yes, find out its degree over Q. (07)

OR

Q-5 a) Check that following polynomials are reducible over Q and R or not. (07)
(1) $f(x) = 18x^2 - 12x + 48$ (2) $f(x) = 2x^2 - 3x + 6$
(3) $f(x) = x^4 + x^3 + x + 1$.

b) $\sqrt{2} . \sqrt{3}$ is algebraic over Q ? If yes, find out its degree over Q. (07)

Q-6 a) If L is a finite extension of K and if K is a finite extension of F then prove L is a finite extension of F. Moreover [ L : F ] = [ L : K ][ K : F ] (07)

b) Prove that subgroup of a solvable group is solvable. (04)

c) Define elementary symmetric functions. (03)

OR

Q-6 a) Prove that $f(x)$ in F[x] has multiple roots if and only if $(f(x), f'(x))$ is constant. (07)

b) Prove that if p(x) is irreducible then <p(x)> is a maximal ideal of F[x]. (05)

c) Define : splitting field. (02)

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